Measuring Length and Area

Geometry

Chapter 11

Geometry 11

This Slideshow was developed to accompany the textbook

- *Larson Geometry*
- *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
- *2011 Holt McDougal*

Some examples and diagrams are taken from the textbook.

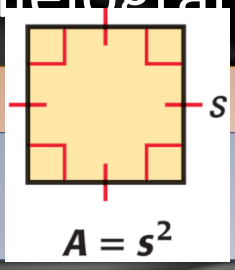
Slides created by
Richard Wright, Andrews Academy
rwright@andrews.edu

11.1 Areas of Triangles and Parallelograms

Area of a Square

$$A = s^2$$

Where s is the length of a side.



Area Congruence Postulate

If 2 polygons are congruent, then they have the same area.

Area Addition Postulate

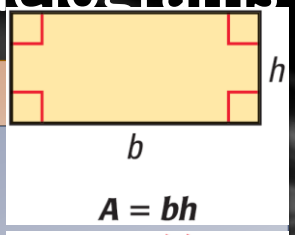
The total area is the sum of the areas of the nonoverlapping parts.

11.1 Areas of Triangles and Parallelograms

Area of a Rectangle

$$A = bh$$

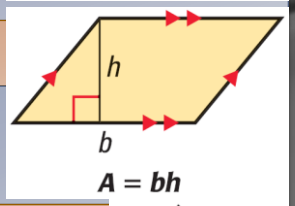
Where b is the base and h is the height.



Area of a Parallelogram

$$A = bh$$

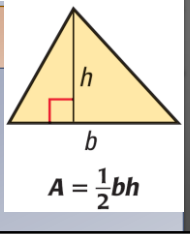
Where b is the base and h is the height.



Area of a Triangle

$$A = \frac{1}{2}bh$$

Where b is the base and h is the height.



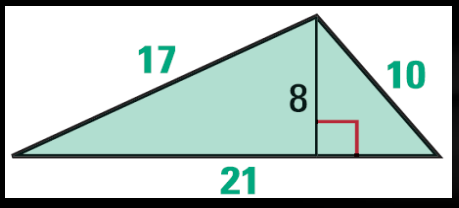
Rectangle can be divided into b by h unit squares.

Parallelogram can be cut apart and built into a rectangle.

Triangle is ½ a parallelogram.

11.1 Areas of Triangles and Parallelograms

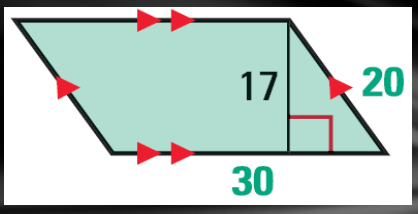
Find the perimeter and area of the polygon.



$$P = 17 + 10 + 21 = 48$$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(21)(8) = 84$$



$$P = 20 + 30 + 20 + 30$$

$$P = 100$$

$$A = bh$$

$$A = 30(17) = 510$$

$$P = 17 + 10 + 21 = 48$$

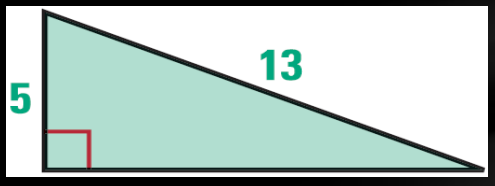
$$A = \frac{1}{2}(21)(8) = 84$$

$$P = 20 + 30 + 20 + 30 = 100$$

$$A = 30(17) = 510$$

11.1 Areas of Triangles and Parallelograms

Find the perimeter and area of the polygon.



Use Pythagorean Theorem to find the other side

$$a^2 + b^2 = c^2$$
$$5^2 + b^2 = 13^2$$
$$25 + b^2 = 169$$
$$b^2 = 144$$
$$b = 12$$
$$P = 5 + 12 + 13 = 30$$
$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(5)(12) = 30$$

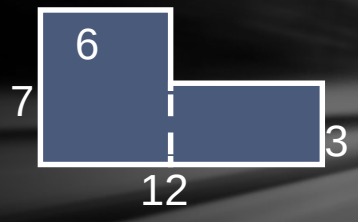
$$a^2 + b^2 = c^2$$
$$5^2 + b^2 = 13^2$$
$$25 + b^2 = 169$$
$$b^2 = 144$$
$$b = 12$$
$$P = 5 + 12 + 13 = 30$$
$$A = \frac{1}{2}(5)(12) = 30$$

11.1 Areas of Triangles and Parallelograms

- A parallelogram has an area of 153 in^2 and a height of 17 in. What is the length of the base?

$$A = bh$$
$$153 = b(17)$$
$$b = 9$$

Find the area.



$$A = bh + bh$$
$$A = 7(6) + 3(6) = 60$$

$$A = bh$$
$$153 = b17$$
$$b = 9$$

Length of right rectangle is 6

$$A = 7(6) + 3(6) = 60$$

Answers and Quiz

11.1 Answers

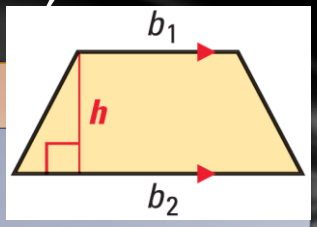
11.1 Homework Quiz

11.2 Areas of Trapezoids, Rhombuses, and Kites

Area of a Trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$

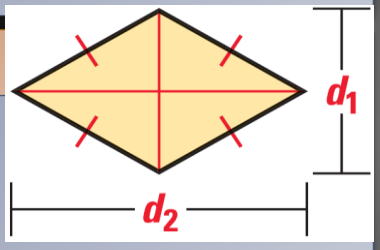
Where h is the height and b_1 and b_2 are the bases.



Area of a Rhombus

$$A = \frac{1}{2}d_1 d_2$$

Where d_1 and d_2 are the diagonals.



Trapezoid is a triangle + parallelogram

$$A = \frac{1}{2}(b_2 - b_1)h + b_1 h$$
$$A = \frac{1}{2}b_2 h - \frac{1}{2}b_1 h + b_1 h$$
$$A = \frac{1}{2}b_2 h + \frac{1}{2}b_1 h$$
$$A = \frac{1}{2}h(b_1 + b_2)$$

Rhombus is four small triangles

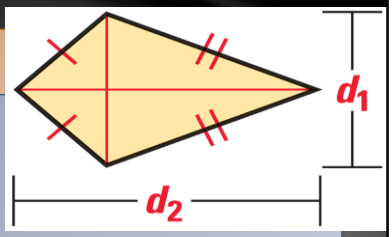
$$A = 4\left(\frac{1}{2}\left(\frac{1}{2}d_1\right)\left(\frac{1}{2}d_2\right)\right)$$
$$A = \frac{4}{8}d_1 d_2$$
$$A = \frac{1}{2}d_1 d_2$$

11.2 Areas of Trapezoids, Rhombuses, and Kites

Area of a Kite

$$A = \frac{1}{2} d_1 d_2$$

Where d_1 and d_2 are the diagonals.



A Kite is two triangles with base d_2 and height ½ d_1

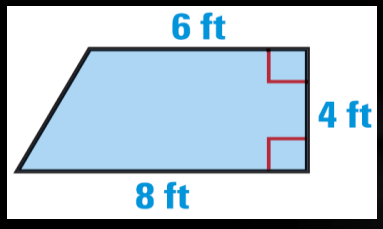
$$A = 2\left(\frac{1}{2}(d_2)\left(\frac{1}{2}d_1\right)\right)$$

$$A = \frac{2}{4} d_1 d_2$$

$$A = \frac{1}{2} d_1 d_2$$

11.2 Areas of Trapezoids, Rhombuses, and Kites

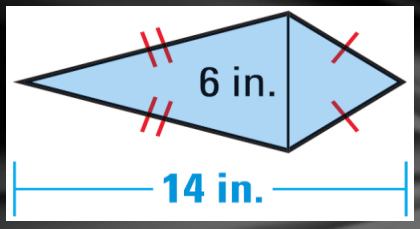
Find the area



Trapezoid

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$A = \frac{1}{2}4(6 + 8) = 28$$



Kite

$$A = \frac{1}{2}d_1d_2$$

$$A = \frac{1}{2}(6)(14) = 42$$

Trapezoid

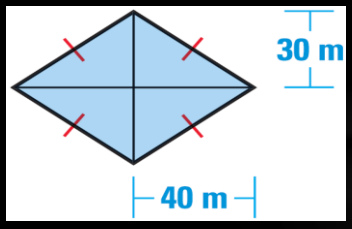
$$A = \frac{1}{2}4(6 + 8) = 28$$

Kite

$$A = \frac{1}{2}(6)(14) = 42$$

11.2 Areas of Trapezoids, Rhombuses, and Kites

Find the area



Rhombus

$$A = \frac{1}{2} d_1 d_2$$

$$A = \frac{1}{2}(80)(60) = 2400$$

Rhombus

$$A = \frac{1}{2}(80)(60) = 2400$$

11.2 Areas of Trapezoids, Rhombuses, and Kites

The area of a kite is 80 ft^2. One diagonal is 4 times as long as the other. Find the diagonal lengths.

$$A = \frac{1}{2} d_1 d_2$$
$$d_1 = 4d_2$$

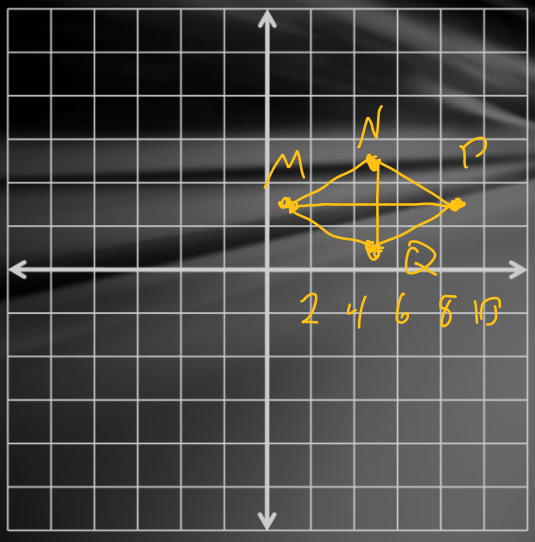
$$80 = \frac{1}{2} 4d_2(d_2)$$
$$80 = 2d_2^2$$
$$40 = (d_2)^2$$
$$\sqrt{40} = d_2$$
$$\sqrt{4}\sqrt{10} = d_2$$
$$d_2 = 2\sqrt{10}$$
$$d_1 = 8\sqrt{10}$$

Kite

$$A = \frac{1}{2} d_1 d_2$$
$$d_1 = 4d_2$$
$$80 = \frac{1}{2} 4d_2(d_2)$$
$$80 = 2d_2^2$$
$$40 = (d_2)^2$$
$$d_2 = 2\sqrt{10}$$
$$d_1 = 8\sqrt{10}$$

11.2 Areas of Trapezoids, Rhombuses, and Kites

Find the area of a rhombus with vertices $M(1, 3)$, $N(5, 5)$, $P(9, 3)$ and $Q(5, 1)$.



Diagonals are $d_1 = 8, d_2 = 4$

$$A = \frac{1}{2} d_1 d_2$$

$$A = \frac{1}{2}(8)(4) = 16$$

Rhombus

Diagonals are $d_1 = 8, d_2 = 4$

$$A = \frac{1}{2}(8)(4) = 16$$

Answers and Quiz

11.2 Answers

11.2 Homework Quiz

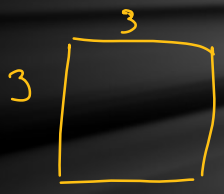
11.3 Perimeter and Area of Similar Figures

What is the perimeter and area of a square that is 1 unit per side?

$P = 4, A = 1$

Triple the sides; what is the perimeter and area of a square that is 3 units per side?

$P = 12, A = 9$



What is the ratio of perimeters?

$\frac{12}{4} = \frac{3}{1}$

What is the ratio of areas?

$\frac{9}{1}$

P = 4; A = 1

P = 12; A = 9

12/4 = 3

9/1 = 9 = 3^2

11.3 Perimeter and Area of Similar Figures

Areas of Similar Polygons

If two polygons are similar with lengths in ratio of $\frac{a}{b}$, then the areas are in ratio of $\frac{a^2}{b^2}$.

11.3 Perimeter and Area of Similar Figures

The perimeter of ΔABC is 16 ft, and its area is 64 ft^2. The perimeter of ΔDEF is 12 ft. Given that ΔABC ~ ΔDEF, find the ratio of the area of ΔABC to the area of ΔDEF.

Lengths $\frac{16}{12}$

Areas $\frac{16^2}{12^2} = \frac{256}{144} = \frac{16}{9}$

Find the area of ΔDEF.

Areas

$$\frac{16}{9} = \frac{64}{A}$$
$$16A = 576$$
$$A = 36$$

Lengths $\frac{16}{12}$

Areas $\frac{16^2}{12^2} = \frac{256}{144} = \frac{16}{9}$

Area of ΔDEF

$$\frac{16}{9} = \frac{64}{A}$$
$$16A = 576$$
$$A = 36$$

11.3 Perimeter and Area of Similar Figures

The ratio of the areas of two regular decagons is 20:36. What is the ratio of their corresponding side lengths in simplest radical form?

$$areas = \frac{a^2}{b^2} = \frac{20}{36}$$

$$lengths = \frac{a}{b} = \frac{\sqrt{20}}{\sqrt{36}} = \frac{\sqrt{4}\sqrt{5}}{\sqrt{36}} = \frac{2\sqrt{5}}{6}$$

Lengths $\frac{a}{b}$

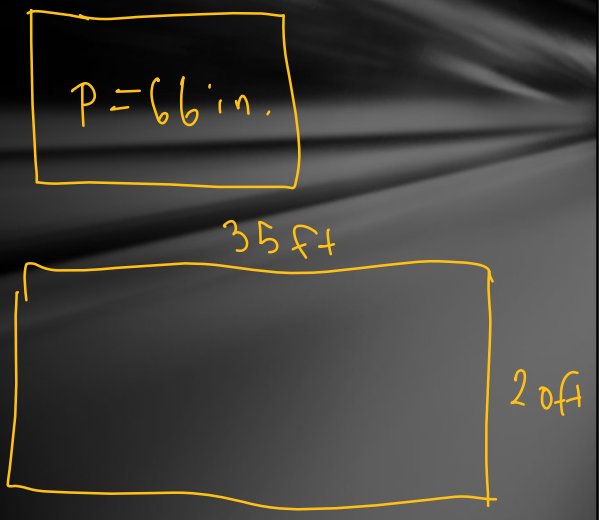
Areas $\frac{a^2}{b^2}$

$$areas = \frac{20}{36}$$

$$lengths = \frac{\sqrt{20}}{\sqrt{36}} = \frac{2\sqrt{5}}{6}$$

11.3 Perimeter and Area of Similar Figures

Rectangles I and II are similar. The perimeter of Rectangle I is 66 inches. Rectangle II is 35 feet long and 20 feet wide. Show the steps you would use to find the ratio of the areas and then find the area of Rectangle I.



Convert 66 inches to feet

$$66in\left(\frac{1ft}{12in}\right) = 5.5ft$$

Find perimeter of Rectangle II

$$P = 35 + 35 + 20 + 20 = 110ft$$

Find ratio of perimeters

$$\frac{5.5}{110} = \frac{1}{20}$$

Find ratio of areas

$$\frac{1^2}{20^2} = \frac{1}{400}$$

Find the area of Rectangle II

$$35(20) = 700ft^2$$

Use the ratio to find the area of Rectangle I

$$\frac{1}{400} = \frac{A}{700}$$

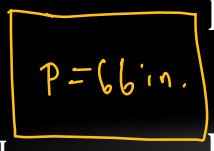
$$400A = 700$$

$$A = \frac{7}{4} = 1.75ft^2$$

11.3 Perimeter and Area of Similar Figures

Convert 66 inches to feet

$$66in\left(\frac{1ft}{12in}\right) = 5.5ft$$



Find perimeter of Rectangle II

$$P = 35 + 35 + 20 + 20 = 110ft$$

Find ratio of perimeters (lengths)

$$\frac{a}{b} = \frac{5.5}{110} = \frac{1}{20}$$

Find ratio of areas

$$\frac{a^2}{b^2} = \frac{1^2}{20^2} = \frac{1}{400}$$

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Answers and Quiz

11.3 Answers

11.3 Homework Quiz

11.4 Circumference and Arc Ler

Circumference of a Circle

- Distance around the circle
- Like perimeter

π

- Ratio of the circumference to the diameter of a circle
- Estimated in 2 Chronicles 4:2 and 1 Kings 7:23 as 3
- 3.141592654…

$$C = \pi d$$
$$C = 2\pi r$$

11.4 Circumference and Arc Length

Find the circumference of a circle with diameter 5 inches.

$$C = \pi d$$
$$C = \pi 5 = 15.7\ in$$

Find the diameter of a circle with circumference 17 feet.

$$17 = \pi d$$
$$\frac{17}{\pi} = d$$
$$d = 5.41\ ft$$

$$C = \pi d$$
$$C = \pi 5 = 15.7\ in$$

$$17 = \pi d$$
$$\frac{17}{\pi} = d$$
$$d = 5.41\ ft$$

11.4 Circumference and Arc Length

A car tire has a diameter of 28 inches. How many revolutions does the tire make while traveling 500 feet?

Convert to the same units

$$28\ in = 2\frac{1}{3}ft$$

$$C = \pi d$$

$$C = \pi\left(2\frac{1}{3}\right) = 7.3304\ ft$$

Each revolution is one circumference

$$Revolutions = \frac{500}{7.3304} = 68.2\ rev$$

$$28\ in = 2\frac{1}{3}ft$$

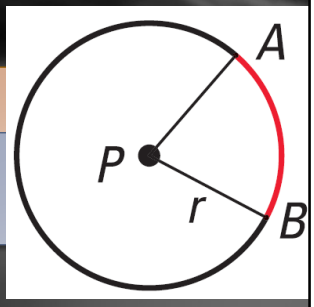
$$C = \pi\left(2\frac{1}{3}\right) = 7.3304\ ft$$

$$Revoultions = \frac{500}{7.3304} = 68.2\ rev$$

11.4 Circumference and Arc Length

Arc Length

- Portion of the circumference that an arc covers



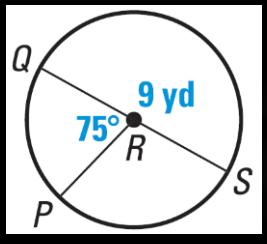
Arc Length

$$\text{Arc Length} = \frac{\text{Arc Measure}}{360°} \cdot 2\pi r$$

$$Arc\ Length\ \ \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360°} \cdot 2\pi r$$

11.4 Circumference and Arc Length

Find the length of $\overset{\frown}{PQ}$.



$$r = 4.5\ yd$$

$$Arc\ Length = \frac{m\overset{\frown}{PQ}}{360°} 2\pi r$$

$$Arc\ Length\ \overset{\frown}{PQ} = \frac{75°}{360°} \cdot 2\pi(4.5\ yd)$$

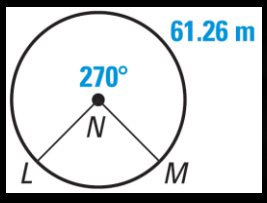
$$= 5.89\ yd$$

$$r = 4.5\ yd$$

$$Arc\ Length\ \overset{\frown}{PQ} = \frac{75}{360} \cdot 2\pi 4.5\ yd = 5.89\ yd$$

11.4 Circumference and Arc Length

Find the Circumference of $\odot N$.



$$Arc\ Length = 61.26\ m$$

$$Arc\ Length = \frac{m\overset{\frown}{LM}}{360°} 2\pi r$$

$$61.26\ m = \frac{270°}{360°} \cdot 2\pi r$$

$$61.26\ m = \frac{3}{4} \cdot 2\pi r$$

$$81.7\ m = 2\pi r = C$$

$$Arc\ Length = 61.26\ m$$

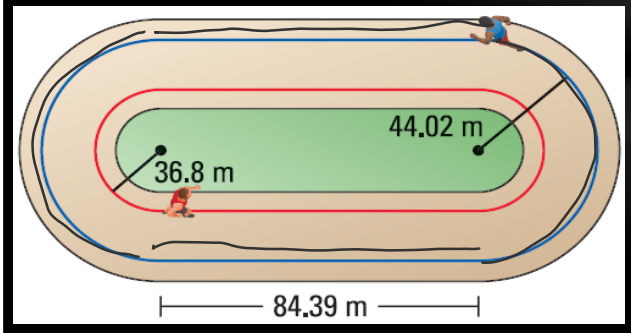
$$61.26\ m = \frac{270}{360} \cdot 2\pi r$$

$$61.26\ m = .75 \cdot 2\pi r$$

$$81.7\ m = 2\pi r = C$$

11.4 Circumference and Arc Length

How far does the runner on the blue path travel in one lap. Round to the nearest tenth of a meter.



The two ends make a circle

$$C = 2\pi r$$

$$C = 2\pi 44.02\ m$$

$$C = 276.59\ m$$

Add the two straight stretches

$$276.59\ m + 2(84.39\ m) = 445.4\ m$$

The two ends make a circle

$$C = 2\pi 44.02\ m = 276.59\ m$$

Add the two straight stretches

$$276.59\ m + 2(84.39\ m) = 445.4\ m$$

Answers and Quiz

11.4 Answers

11.4 Homework Quiz

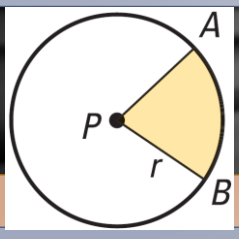
11.5 Areas of Circles and Sectors

Area of a Circle

$$A = \pi r^2$$

Sector of a Circle

- Fraction of a Circle



Area of a Sector

$$A = \frac{\text{Arc Measure}}{360°} \cdot \pi r^2$$

11.5 Areas of Circles and Sectors

Find area of $\odot D$

$$A = \pi r^2$$

$$A = \pi(14)^2 = 615.8\ ft^2$$

Find area of red sector

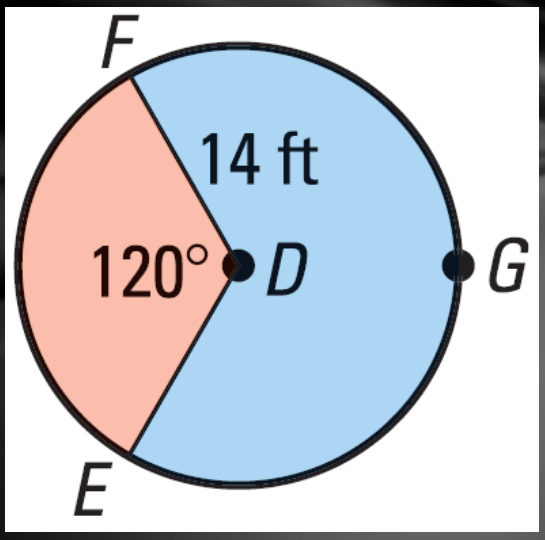
$$A = \frac{Arc}{360°}\pi r^2$$

$$A = \frac{120°}{360°}\pi 14^2 = 205.3\ ft^2$$

Find area of blue sector

$$A = \frac{Arc}{360°}\pi r^2$$

$$A = \frac{240°}{360°}\pi 14^2 = 410.5\ ft^2$$



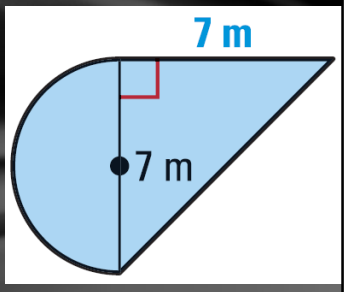
$$A = \pi 14^2 = 615.8\ ft^2$$

$$A = \frac{120}{360}\pi 14^2 = 205.3\ ft^2$$

$$A = \frac{240}{360}\pi 14^2 = 410.5\ ft^2$$

11.5 Areas of Circles and Sectors

Find the area of the figure.



Semicircle

$$A = \frac{1}{2}\pi r^2$$

$$A = \frac{1}{2}(\pi(3.5)^2) = 19.2423\ m^2$$

Triangle

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(7)(7) = 24.5\ m^2$$

Total

$$19.2423\ m^2 + 24.5\ m^2 = 43.7\ m^2$$

Semicircle

$$A = \frac{1}{2}(\pi 3.5^2) = 19.2423\ m^2$$

Triangle

$$A = \frac{1}{2}(7)(7) = 24.5\ m^2$$

Total

$$19.2423\ m^2 + 24.5\ m^2 = 43.7\ m^2$$

Answers and Quiz

11.5 Answers

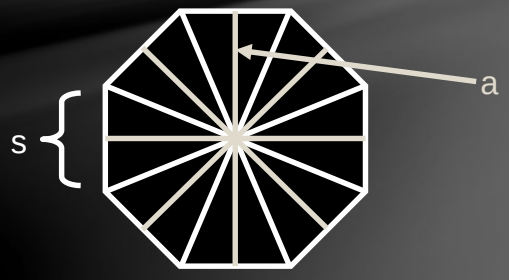
11.5 Homework Quiz

11.6 Areas of Regular Polygons

Now that we know how to find the area of a triangle we can find the area of any polygon since it can be broken up into triangles.

For example find the area of a stop sign.

$$A = \frac{1}{2}Pa$$

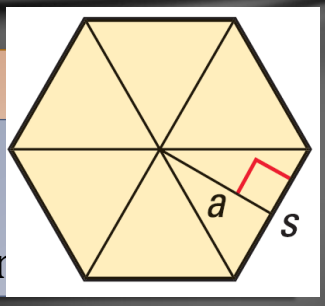


It has 8 sides, I'll call them *s*.
If we draw lines connecting opposite vertices, we have 8 identical triangles.
Draw the altitudes from the center of the sign and call it *a*.
The area of each triangle is ½ *sa*.
The area of the sign then is 8(½ *sa*).
But the perimeter, *P*, is 8*s,* so the Area = ½ *Pa*.

11.6 Areas of Regular Polygons

Apothem

- A segment drawn from the center of a regular polygon perpendicular to the edge (also bisects edge)



Area of a Regular Polygon
$A = \frac{1}{2}Pa$
Where P is the perimeter and a is the apothem

11.6 Areas of Regular Polygons

Typical steps to find area of regular polygon

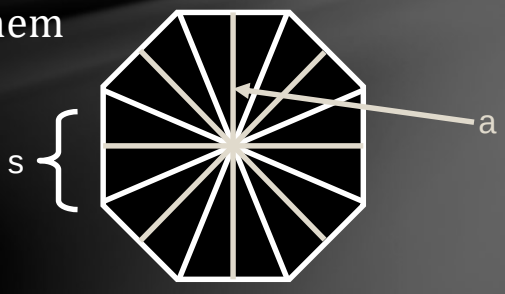
Find ½ of central angle

- $\frac{1}{2}\left(\frac{360}{n}\right)$

Use trigonometry to find apothem

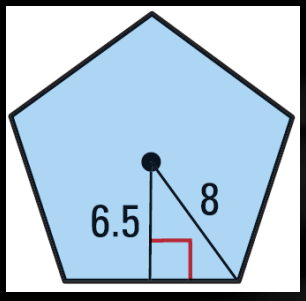
- tan, sin, cos

$A = \frac{1}{2}Pa$



11.6 Areas of Regular Polygons

- Find the area of the regular polygon.



- Pythagorean theorem to find side

$$6.5^2 + x^2 = 8^2$$
$$42.25 + x^2 = 64$$
$$x^2 = 21.75$$
$$x = 4.6637$$
$$s = 2x = 9.3274$$

- Area

$$A = \frac{1}{2}(9.3274 \cdot 5)(6.5) = 151.6$$

Pentagon

- Pythagorean theorem to find side

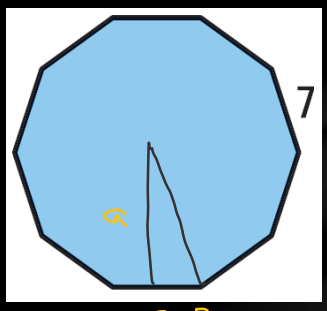
$$6.5^2 + x^2 = 8^2$$
$$42.25 + x^2 = 64$$
$$x^2 = 21.75$$
$$x = \frac{\sqrt{87}}{2} = 4.6637$$
$$s = 2x = 9.3274$$

- Area

$$A = \frac{1}{2}(9.3274 \cdot 5)(6.5) = 151.6$$

11.6 Areas of Regular Polygons

- Find the area of the regular polygon.



- Find ½ central angle

$$\frac{1}{2}\left(\frac{360}{10}\right) = 18°$$

- Find apothem

$$\tan 18° = \frac{3.5}{a}$$
$$a \cdot \tan 18° = 3.5$$
$$a = 10.7719$$

- Find area

$$A = \frac{1}{2}(7 \cdot 10)(10.7719) = 377.0$$

Decagon

- Find ½ central angle

$$\frac{1}{2}\left(\frac{360}{10}\right) = 18°$$

- Find apothem

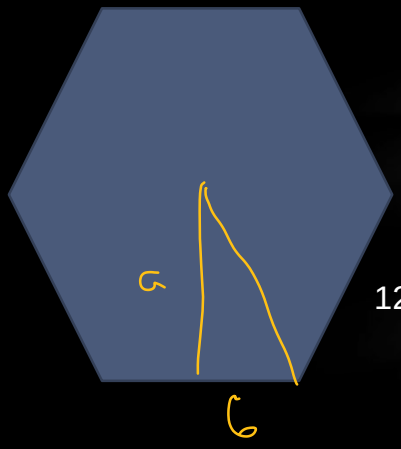
$$\tan 18° = \frac{3.5}{a}$$
$$a \cdot \tan 18° = 3.5$$
$$a = 10.7719$$

- Find area

$$A = \frac{1}{2}(7 \cdot 10)(10.7719) = 377.0$$

11.6 Areas of Regular Polygons

Find the area.



$$\frac{1}{2}side = 6$$

$$\frac{1}{2}central\ angle = \frac{1}{2}\left(\frac{360°}{6}\right) = 30°$$

Apothem $\tan 30° = \frac{6}{a}$

$$a \cdot \tan 30° = 6$$

$$a = 10.3923$$

Area $A = \frac{1}{2}Pa$

$$= \frac{1}{2}(12 \cdot 6)(10.3923) = 374.1230$$

Hexagon:

$$\frac{1}{2}side = 6$$

$$\frac{1}{2}central\ angle = \frac{1}{2}\left(\frac{360}{6}\right) = 30$$

Apothem $\tan 30 = \frac{6}{a}$

$$a \cdot \tan 30 = 6$$

$$a = 10.3923$$

Area $\frac{1}{2}(12 \cdot 6)(10.3923) = 374.1230$

Answers and Quiz

11.6 Answers

11.6 Homework Quiz

11.7 Use Geometric Probability

Let's say you are listening to a radio contest where you hear a song and call in and name it.

- The song was supposed to be played between 12:00 and 1:00, but you can only listen from 12:20 to 1:00 because that is when you get out of class.
- What is the probability that you will hear the song?

$$\text{Probability} = \frac{\text{Favorable Outcomes}}{\text{Total Outcomes}}$$

- But we have basically a line (timeline), so Probability will be $\frac{40\ min}{60\ min} = \frac{2}{3} \approx 67\%$

11.7 Use Geometric Probability

Length Probability Postulate

If a point on AB is chosen at random and C is between A and B, then the probability that the point is on AC is (Length of AC)/(Length of AB).

$$P(AC) = \frac{AC}{AB}$$

11.7 Use Geometric Probability

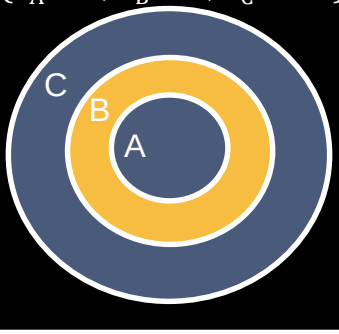
Area Probability Postulate

If a point in region A is chosen at random, then the probability that the point is in region B, which is in the interior of region A, is (Area of region B) / (Area of region A)

$$P(B) = \frac{Area\ of\ B}{Area\ of\ A}$$

11.7 Use Geometric Probability

Joanna designed in a new dart game. A dart in section A earns 10 points; a dart in section B earns 5 points; a dart in section C earns 2 points. Find the probability of earning each score. Round to the nearest hundredth. ($r_A = 2$, $r_B = 5$, $r_C = 10$)



Area of $A = \pi 2^2 = 12.566$

Area of $B = \pi 5^2 - 12.566 = 65.974$

Area of $C = \pi 10^2 - \pi 5^2 = 235.619$

Area of Board = $\pi 10^2$ = 314.159

$$P(A) = \frac{A}{Total} = \frac{12.566}{314.159} = .040 = 4\%$$

$$P(B) = \frac{B}{Total} = \frac{65.974}{314.159} = .21 = 21\%$$

$$P(C) = \frac{C}{Total} = \frac{235.619}{314.159} = .75 = 75\%$$

Area of A = $\pi 2^2$ = 12.566
Area of B = $\pi 5^2$ – 12.566 = 65.974
Area of C = $\pi 10^2$ – $\pi 52$ = 235.619
Area of Board = $\pi 10^2$ = 314.159
P(A) = 12.566/314.159 = .040 = 4%
P(B) = 65.974/314.159 = .21 = 21%
P(C) = 235.619/314.159 = .75 = 75%

Answers and Quiz

11.7 Answers

11.7 Homework Quiz